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A Modal Approach to Sneed's "Theoretical Functions"

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The aim of this paper is to argue that modal logic, besides (informal) set theory and category theory, bears certain relevance to the study of Sneed's ideas. In the first part of my paper I try to provide a modal reconstruction of Sneed's definition of "theoretical finctions" and in the final one the machinery of modal semantics is used in discussing some arguments to the effect that at least in certain contexts laws and constraints should be closely related. J. D. Sneed did in correspondence agree with the way I dealt with constraints in reconstructing theoretical functions as modal concepts; but he has also expressed his pessimism concerning the possibility that my treatment of constraint as a generalized law could be managed so that to explain the transport of information across different potential models of a theory T. I do not think that is a fatal objection; but I also think it deserves a special and detailed analysis.

I. *The modal criterion of the "theoretical"*. A constraint C for a function f of T is defined in Sneed (1977) as a family of sets of potential models of T such that: (i) $\emptyset \notin C$; (ii) if i is a potential model, then $\{i\} \in C$; (iii) if X is in C and $Y \subseteq X$, then Y is in C. (I suppose henceforth that f is a theoretical function in Sneed's sense). It is not difficult to see that constraint C could be conceived of as a binary relation on the set Mp (of all the potential models of T) if we let $C(i, j)$ hold iff there is an X in C and $i \in X$, $j \in X$. Now by definition $\{i\} \in C$ for each $i \in Mp$ and therefore $C(,)$ is reflexive; and obviously, if $C(i, j)$ holds, then $C(j, i)$ also does. Therefore, C is a reflexive and symmetrical relationship. However, constraints in general are not identifiable with binary relations and most interesting constraints seem to be n-ary relations, with $n > 2$. in this paper I shall explore the case when constraints are just binary relations. My preference is based on the fact that in this case the modal reconstruction of Sneed's "theoretical functions" could be designed by means of a well-known modal logic: the so-called Brouwerian system (Kripke 1963). The semantic formalism seems to generalize naturally to the other cases; but the detection of the appropriate axiomatization of the logics obtained when n-ary "alternativeness" relations among "possible worlds" are taken into account is much more difficult.

It is assumed that the reader is roughly acquainted with standard (Kripke-type) modal semantics. A model for the Brouwerian system is a quadruple $S = (K, R, U, V)$ such that: K is a non-empty set (its members are often called "possible worlds"); R is a symmetrical and reflexive relation on K; U is the set of all the (admissible) individuals; and V is a function such that: (i) if $w \in K$, then $V(w) \subseteq U(V(w)$ is regarded as the set of all the individuals existing in w); (ii) if a is an individual constant, then $V(a, w) \in U$; if P^n is an n-ary predicate letter, then $V(P^n, w) \subseteq \subseteq V(w)^n$; $V(P(a_1 \ldots a_n), w) = 1$ if $(V(a_1, w) \ldots V(a_n, w)) \in V(P^n, w)$ and $V(P(a_1 \ldots a_n), w) = 0$ if $(V(a_1, w) \ldots V(a_n, w)) \in V(w)^n$ and $(V(a_1, w) \ldots V(a_n, w)) \notin V(P^n, w)$; otherwise $V(P(a_1 \ldots a_n), w)$ is not defined. The truth assignment conditions for compound expressions as $-A$, $A \vee B$, $(\mathrm{E}x)A(x)$ or $\Box A$ go as usual. However, have a moment's reflection on definition of V when its first argument is a modal sentence such as "$\Box A$". The standard condition is:

$V(\Box A, w) = 1$ if $V(A, w') = 1$ for all w' such that $R(w, w')$ and $V(\Box A, w) = 0$ if $V(A, w') = 0$ for some w' such that $R(w, w')$; otherwise $V(\Box A, w)$ is not defined.

Now we should avoid truth-value gaps and to do that we need the following condition (the Kripkean condition – (Kripke 1963a)):

$$\text{If } R(w, w'), \text{ then } V(w) \subseteq V(w') \tag{1}$$

Let Δ_i be the domain of the potential model i of theory T: $i \in Mp(T)$, and let $j \in Mp(T)$. The modal reconstruction of Sneed's theoretical functions is based on the claim that constraint C plays within T the job relation R enjoys and that potential models behave exactly as we usually assume that possible worlds do. Perhaps the only reasonable alternative to think about "possible worlds" is to conceive them of as potential models of our theories concerning the world.

But there is no reason why for all i and j if $C(i, j)$, then $\Delta_i \subseteq \Delta_j$ (some grounds for rejecting the Kripkean condition are also to be found in some special, e.g. temporal, modal logics). That is why I shall slighty modify the semantics of the Brouwerian logic: the Kripkean condition is deleted and consequently a new definition is provided for function V in the case when its first argument is modal[1].

$V(\Box A, w) = 1$ iff $V(A, w) = 1$ and for all w', if $R(w, w')$, then $V(A, w') \neq 0$ (i.e., $V(A, w') = 1$ or $V(A, w')$ is not defined). "Possibility" is defined as usual: $\Diamond A = \text{df.} - \Box - A$. Then:

$V(\Diamond A, w) = 1$ iff $V(A, w) = 1$ or there is a w' such that $R(w, w')$ and $V(A, w') = 1$.

Now if R is reflexive (and that is the case when the Brouwerian modal logic is concerned) then the above definition reduces to the standard one:

1 There it is also necessary to modify the definition of validity: A is valid at $S = (K, R, U, V)$ iff there is a $w \in K$ such that $V(A, w) = 1$ and for each $w' \in K$, $V(A, w') \neq 0$. A is B-valid iff for all models S of the Brouwerian system, A is valid at S.

$V(\Diamond A, w) = 1$ iff there is a w' such that $R(w, w')$ and $V(A, w') = 1$.

A proposition $^\wedge A$ is usually identified with a set of possible worlds and $^\wedge A$ is true at w iff $w \in {}^\wedge A$. Propositions could be divided into two sorts: modal and non-modal. A proposition is regarded to be non-modal-at-world-w if its truth-value at w is simply determined by looking at facts in w. The proposition expressed by the sentence "It rained in Bucharest on March 10,1984" is true at the actual world if it really rained in Bucharest on March 10,1984. A proposition is modal-at-world-w (or: w-modal) if fixing its truth-value at w needs fumbling in some (other) worlds w', w'', w''' and looking at facts in those worlds. The proposition expressed by the sentence "It is possible that George would buy a new car" is true at the actual world if there is some world w' which is possible relative to ours and facts in w' are so that it is the case at w' that George bought a new car. I shall say that a proposition is relatively-modal iff there is some world w such that it is w-modal; and that it is absolutely-modal iff for all w it is w-modal. Standard (modal) semantics does not distinguish between relatively or absolutely modal (or, analogously, non-modal) propositions: if a proposition is relatively-modal (or, non-modal), then, according to standard (one-dimensional) semantics, it is absolutely modal (or, non-modal) too. However, the difference is to be met with n-dimensional modal semantics.

Let me state now Sneed's (or, better to say, a Sneed-like) modal criterion of the theoretical.

If a proposition $^\wedge A$ refers to relationships defined by means of the theoretical and/or non-theoretical functions of a theory T, then $^\wedge A$ might be identified with a set of potential models of T. If $i \in Mp$, then $i \in {}^\wedge A$ iff the relationships referred to by $^\wedge A$ hold at i. $^\wedge A$ is called a T-proposition iff $^\wedge A \subset Mp(T)$.

Let $N = (Mp, Mpp, r, M, C)$ be a core for theory T.

The best-known reconstructions of physical theories carried out by use of Sneed's formalism are due to Sneed himself – the reconstruction of classical particle mechanics (*CPM* henceforth) (Sneed 1971) –and to C. U. Moulines (Moulines 1975) – the reconstruction of classical thermodynamics. In this paper I shall mainly refer to *CPM* and to some of its subtheories: *MPM*, *NCPM* and others. (Sneed, 1971, ch. V). A remarkable feature of such theories, which make them differ from e.g. *CPM*, is that no special laws and constraints for theoretical and/or non-theoretical functions are required. In Balzer and Sneed (1977–1978) a new approach to theories is considered. A theory such as *CPM* is defined as a net-theory made up of interrelated element-theories such as *MPM*, *NCPM*.

As far as *CPM* is concerned, the only constraints on its theoretical function m are independence of system and extensivity (with respect to the "concatenation" operation).

If $^\wedge A$ is a necessary proposition at world w, then $^\wedge A$ is true at world w and if $R(w, w')$ holds, then $^\wedge A$ could not be false at w'. Analogously, if the *CPM*-proposition expressed by the sentence "$m(a) = k$" is true at i, then – as Sneed claims – it could not be false at any (potential) model j such that $C(i, j)$ holds. Thus, that *CPM*-proposition is a necessary one. Nevertheless, the proposition expressed by "$m(a) = k$" needs not be true (and consequently necessary) at j. Indeed, if $a \notin \Delta_j$, it has no truth-value at j. Note also that the proposition expressed by "$(Ea)(m(a) = k)$" could be true, but not necessary at i; and the proposition expressed by "$s(a) = (x_1, x_2, x_3)$" i which occurs the *CPM*-non-theoretical function s could be true, but not necessaar at i. Let f_i be a concrete function (subsumed under the abstract function f) occurring in the i-th a application T_i of T. Suppose a measurement $f_i(a)$ for an object $a \in \Delta_i$ is T-dependent, i.e. if $i \in Mp$, then there is an $a \in \Delta_i \cap D_I(f_i)$ such that in each exposition of T_i the descriptions of a method of measuring $f_i(a)$ contain a sentence in which (another) $j \in Mp$ is involved (Stegmüller, 1976, p. 45). Then, the T- proposition expressed by the sentence "$f_i(a) = k$" is i-modal.

Now the *modal criterion of the theoretical* is the following:

1. A function f is T-dependent at $i \in Mp$ if there is a T-proposition $^\wedge A$ and: (i) f is the only non-logical function occurring in $^\wedge A$; (ii) $^\wedge A$ is i-modal; (iii) there is a *model* $j \in M$ and $C(i, j)$ and $^\wedge A$ is true at j (i.e., $j \in {}^\wedge A$).

2. A function f is T-theoretical iff for each i, f is T-dependent at i.

According to Sneed (1971) it is always sufficient to take into account T-propositions expressed by sentences like "$m(a) = k$" (in the case of *CPM* at least). Such T-propositions are absolutely-modal ones, i.e. for each i, they are i-modal. Therefore, the Sneedian definition of the theoretical functions could be simplified:

A function f is T-theoretical iff: (i) there is a T-proposition $^\wedge A$ in which f is the only non-logical function and $^\wedge A$ is absolutely-modal; (ii) for each $i \in Mp$ there is a model $j \in M$ and $C(i, j)$ and $^\wedge A$ is true at j.

It is important to note that $^\wedge A$'s being modal at i does depend on its structure and not on the cross-connexions C between elements of Mp. Constraints C only show which are the models and potential models of T we have to look at in the attempt to determine if $i \in {}^\wedge A$ or $i \notin {}^\wedge A$.

One of the constraints on function m at *CPM* is that if $a \in \Delta_i \cap \Delta_j$, then $m_i(a) = m_j(a)$. Let $X \in C$, $Y \in C$, $i \in X$, $j \in X$, $j' \in Y$, $j \in Y$, $a \in \Delta_i$, $a \in \Delta_{j'}$. Suppose sentence "$m(a) = k$" expresses a true (and also, accord- to Sneed, a necessary) *CPM*–proposition at i and that sentence "$m(a) = k'$" ($k \neq k'$) expresses a true (and also a necessary) *CPM*-proposition at j'. The *CPM*-proposition expressed by "$m(a) = k$" is not false at j, for $C(i, j)$ holds. The *CPM*-proposition expressed by "$m(a) = k'$" is not false at j, for

$C(j, j')$ holds. Both of them have no truth-value at j. Obviously, that is the case when $a \notin \Delta_j$.

It seems, however, that the model criterion of the theoretical I proposed above needs some further comments. It is condition (liii): "there is a *model* $j \in M$ and $C(i,j)$ and $\wedge A$ is true at j" which makes us sure that the distinction between theoretical and non-theoretical functions is T-dependent and does not depend only on class Mp and constraints C on Mp. Second, it highlights Sneed's idea that a measurement of a concrete function f_i is T-dependent iff there is an object a in the domain Δ_i of i so that in all existing expositions of application i of T all methods of measuring $f_i(a)$ presuppose that some application of T is successful (Sneed, 1971, p. 31). This appeal to a "*successful application*" is referred to by the claim that j is a *model* and not only a member of Mp.[2]

And yet the claim that $j \in M \subset Mp$ would puzzle the modal logician: for he would accept that "possible worlds" are to be divided into two sorts, but he still maintains that the only criterion lies in the alternativeness relation R. For each $w \in K$, all the "worlds" in K are either "possible relative to" w – i.e. $R(w, w')$ holds – or they are not – i.e. $R(w, w')$ does not hold. Some new developments are therefore needed. First, he could distinguish "normal" from "non-normal worlds", as he did when dealing with C. I. Lewis's modal systems S1–S3, which would be supposed to cover the distinction between successful and unsuccessful applications of a theory T. Second, he has at hand another alternative. He only needs to replace "model" by "potential model" in condition (liii) above and add two special requirements on the core $N = (Mp, Mpp, r, M, C)$:

If i, i' are potential models, then there is another potential model j such that $\Delta_j = \Delta_i \cup \Delta_{i'}$. (2)

For each $i \in Mp$ there is an $j \in M$ such that $\Delta_i = \Delta_j$. (3)

(2), according to W. Stegmüller, is supposed to hold (Stegmüller, 1976, p. 164). (3), on the other hand, seems to be a necessary condition for a person's p having a theory T. If p has T, then he maintains that T could be successfully applied to each domain Δ, provided that the application is not (or seems not to be) meaningless. Now, if one agrees with (3), then by the above definition of the "theoretical" (appropriately modified) he is endowed with a *normal* modal reconstruction of the "theoretical" (for, indeed, "having" or "holding" a theory T is claimed to be a rational reconstruction of Kuhn's "normal science").

2 Note also that the modal reconstruction of the "theoretical" I proposed in this paper is based on some particular assumptions. It could e.g. be refined so that the requirement that only T-propositions in which just one non-logical function occurs should be taken into account be deleted, or it could be refined in other ways.

It should also be emphasized that (2), (3) and condition: "there is a potential model j and $C(i, j)$ and $^\wedge A$ is true at j" together imply condition (liii) in the above definition of a theoretical function, but the converse does not hold.

According to W. Stegmüller (1976), Sneed's criterion of the "theoretical" is *absolute* and functional. But as we have seen it relies on a sharp cut of successful from unsuccessful applications of T. His statement assumed that the cut is not theory-dependent, that we could choose models from potential models of T whatever T's theoretical resources would be. If, on the other hand, we ask that (3) should hold and therefore that one is concerned with a normal definition of a theoretical function, being a successful application, i.e. a model of T, is not involved directly in the definition. It is rather a property core N has (recall that N is an essential element in Sneed's definition of a theory). By (3), a close connection between *Mp*'s and *M's is provided:* it highlights an intratheoretical relation, and not a relation obtaining between a theory T and some extra data.

"Holding a theory" is, of course, a pragmatic concept; but I believe that the normal definition of the theoretical functions is still a semantical matter.

II. *How to law a constraint.* In the first part of my paper I dealt with constraints only in so far as they were involved in the modal reconstruction of theoretical functions. In that context it was necessary to assume that constraints express cross-connexions between different *Mp*'s. Now I turn to another significant context: the *theoretization relation.* The point is exactly this: how should we describe their work in the movement from a theory T to another theory T' such that T' is a theoretization of T? I believe that in this case the sharp distinction of constraints from laws is misleading and that in the theoretization context there is no intuitive way to put apart constraints from laws. One argument is sketched below; but the conception of constraints and laws as generalized (n-dimensional) laws also entails some other reasons to the same effect.

Nevertheless, on might be tempted to think that my arguments are designed to reduce constraints to laws. That would be a misunderstanding, however. On the one hand, I argue that constraints and laws are distinct except in some context, i.e. the theoretization relation. On the other hand, I do not try to reduce constraints to standard laws (laws in Sneed's sense = sets of potential models). I rather argue that laws besides constraints should be regarded as generalized laws.

Now the argument runs as follows: let T' be a theoretization of T. The T-models are viewed as T'partial potential models. (In particular, *CPM*-partial potential models are reconstructed in Sneed (1971) as *PK*-

models. In the general case, Sneed conjectures that each *T*-potential model could be identified with a *T'*-partial potential model. It seems that for some purposes, especially when the reduction relation is approached, this claim is indeed correct. But we do not usually think of theoretization as of a sort of reduction between theories. Rather, if *T'* is a theoretization of *T*, we postulate that *T'* presupposes *T*. We assume that some of *T*'s applications are successful and wish to apply *T'* exactly to those domains which are successful applications of *T*; but if that is not the case, the attempt would be odd. Note that the argument is based on the theory-hierarchy hypothesis). *T*-models are enriched to *T'*-potential models by adding *T'*-theoretical functions. *T'*-constraints are *n*-ary relationships on the set of *T'*-potential models. The question we need for an answer is the following: how are *T*-constraints to be conceived of from the point of view of *T'*? A tentative answer would be that *T*-constraints are *n*-ary relationships on the set of *T'* 's potential models; or to put it in other words, constraints on non-theoretical functions of a theory are formally analogous to constraints on its theoretical functions. But that is untenable.

T-constraints are relations on the set of all *T*'s potential models. If the theoretization relation is not trivial, the set of *T*'s potential models does not intersect with the set of *T'*'s potential models. Suppose, however, that there would be a certain homomorphism between the two sets so that each *T*-potential model is mapped on a certain *T'*-potential model. But each *T*-model could be enriched in different non-equivalent ways to *T'*-potential models; and second – that is a fatal objection on my view – a *T'*-potential model is an enrichment of a certain *T*-model, not of a *T*-potential model. And yet constraints are relationships on the whole set of the potential models of a theory!

Constraints on *T'*-non-theoretical functions could not then be defined as *n*-ary relationships among *T'*-potential models. On the other hand, *T'* presupposes *T* and its core does somehow contain constraints on *T*-theoretical functions. Therefore, the problem is exactly this: how does *T'* contain those constraints? As concerns the set of *T*'s potential models, they are cross-connexions on it. What about *T'*? My hypothesis is that they are internalized into the very structure of each *T'*-potential model: they are elements in the inner structure of each potential model of *T'*.

I think that some insight is gained by letting laws and constraints be unified as generalized propositions. That is why I regard modal logic as a interesting approach to Sneed's ideas.

If *C* is a relationship on *Mp*, it could be identified with a set of pairs of elements of *Mp*. Thus, a sentence like "Mass is an extensive quantity" does not express a *CPM*-proposition on the standard account in modal semantics. Laws, as defined in Sneed (1971, p. 179) are sets of potential models while constraints are sets of pairs of potential models. They are formally heterogenous. Laws are T-propositions while constraints are not T-propositions.

Sneed defines a constraint C as a structured set of non-empty sets $X \subsetneq Mp$. Now, each X is a T-proposition and one might be tempted to identify C with the conjunction $\wedge X_k$ (for all X_k in C); and yet that alternative is not workable, for 1) C might be an infinite set; and 2) $\wedge X_k$ is always the null-set. Indeed, if i and j are potential models, $\{i\} \in C$ and $\{j\} \in C$; but $\{i\} \cap \{j\} = \emptyset$ and consequently $\wedge X_k = \emptyset$.

However, some writers tried to develop, on both logical and philosophical grounds, another account in modal semantics: (Strawsonian) bi-dimensional semantics. As van Fraassen states, the intuitive idea is that both context and facts are parts of a possible world. Therefore, "the world determines first what proposition is expressed, and then whether the proposition is true or not" (van Fraassen, 1977, p. 76).

Consequently, a proposition $\hat{}A$ might be identified with a set of pairs of possible worlds: $(w, w') \in \hat{}A$ iff what sentence A expresses at w is true at w'. Let $\hat{}A(w)$ be what A expresses at w; then $\hat{}A(w)$ is a set of possible worlds and $w' \in \hat{}A(w)$ iff $\hat{}A(w)$ is true at w'.

Turn now to T-propositions. A T-proposition is to be identified with a set of pairs of elements of Mp. Suppose T is *CPM*; the *CPM*-proposition expressed by the sentence "Mass is an extensive quantity" is, on the bi-dimensional account, a T-proposition, i.e. a set of pairs $(i, j) \in Mp \times Mp$. The intuitive (Strawsonian) idea is that the meaning of A varies when moving from one to another application of *CPM*: if $i \neq j$, then probably $\hat{}A(i) \neq \hat{}A(j)$. Therefore, $\hat{}A$ is not a context-free *CPM*-proposition:

$\hat{}A(i) = \text{df. } \{j\text{: what } A \text{ expresses at } i \text{ is true at } j\}$

However, it seems possible to find out some T-propositions $\hat{}B$ so that for each i and j in Mp:

$\hat{}B(i) = \hat{}B(j)$

These are T-propositions on the standard account and Sneed identifies them with laws.

Suppose, on the other hand, that X is a law, i.e. $X \subsetneq Mp$. It is always possible to expand X in a genuine natural way to a T-proposition X' in the sense of bi-dimensional semantics as follows:

$(i, j) \in X'$ iff $j \in X$.

Obviously, $(i, j) \in X'$ iff $(i', j) \in X'$: for all j, what X' is at i is true at j iff what X' is at i' is true at j, i.e. $X'(i) = X'(i')$ for all i, i' in Mp.

To conclude: laws and constraints are formally homogeneous; constraints no less than laws are T-propositions, e.g., in the bi-dimensional case, sets of pairs of potential models.

One second observation I want to do is this. Sneed and Stegmüller seem to share a deep and hidden Platonistic view; they assume that laws (defined on a frame $F = \langle Mp, Mpp, r, M \rangle$) are context-free entities, that they do not depend essentially on particular applications. Formally, their point can be stated as follows:

If $^\wedge A$ is a law, then $^\wedge A(i) = {}^\wedge A(j)$ for all i, j in Mp or, as Sneed makes the point: laws are subsets of Mp.

Are laws standard (one-dimensional) T-propositions? I believe they are not; but to argue for such a position needs very detailed analyses of particular physical theories (e.g., of *CPM* or classical thermodynamics). Therefore, here I shall only sketch it. The definition of laws Sneed advanced seems to ignore the existence of paradigmatic applications of theories. For let i and j be two intended applications of a theory T. Then laws-at-i and laws-at-j might be not identical: they are application-laden. So, if X is a law, X's being law-at-i might be not identical with X's being law-at-j.

Nevertheless, Sneed's formalism involves at a crucial point that view: he argues that there are situations in which we appear to assume, postulate, hypothesize (thus our claim is not empirical!) that theoretical functions have some special form in certain applications. The example of Newton's second law is most frequently cited: "Newton showed that any particle, whose path is a conic section and whose motion along that path obeys Kepler's second law, must be acted upon by a resultant force directed toward one focus of the conic whose magnitude is inversely proportional to the square of the particle's distance from this focus"... If the motion of such a particle is to provide a model for *CPM* whose crucial axiom is roughly $f = m \cdot a$, then the force function f must have a particular form (Sneed 1971, pp. 98–99). Sneed concedes that "all claims of this sort must be satisfactorily accounted for in a *logical reconstruction*" and he himself provides two such logical reconstructions. In Sneed (1971) he introduced different restrictions of the same basic predicate, determining a different special form of the theoretical facts, i.e. special laws and constraints are introduced. In Balzer and Sneed (1977–1978) these claims are reconstructed by use of the notions of element-theory and net-theory.

I believe that the conception of laws as T-propositions brings forth a third reconstruction. That not all laws could be identified with trivial generalizations of standard (Sneedian) laws is, on my view, a very sound argument for that (moreover, I believe that these are the most interesting laws). Strictly speaking, bi-dimensional semantics is not of very much help. There are situations in which three-dimensional laws are necessary and also require constraints to be reconstructed as four-dimensional T-propositions (while most laws are identified with bi-dimensional ones).

A more general semantics is asked. In Miroiu (1984) I have tentatively proposed a local (pseudo-Kantian) semantics with a view to accomodating such situations where n-dimensional propositions (with a varying n) are involved[3].

III. *T-laws.* If X is a law on a frame $F = (Mp, Mpp, r, M)$, then on the standard account, X is a T-law iff $M \subset X$: X is a T-law iff it is true at each model i of the theory T. What about constraints? They are necessary elements in the core N of T, but they are not involved in the definition of a frame F and thus in the definition of a T-law. Thou puzzling, this need not be surprising, for constraints are relations on Mp, while laws are subsets of Mp. On the contrary, within the frame of bi-dimensional semantics laws and constraints are formally identical; therefore, the very definition of laws and constraints should essentially involve that.

I follow here B. van Fraassen (van Fraassen, 1977).

I say that X is a generalized law for a core (not a frame!) $N = (Mp, Mpp, r, M, C)$ iff there is a T-proposition $\hat{}A$ (a set of pairs of Mp's) and X is $\hat{}(C \rightarrow A)$. The semantic condition is this:

$(i, j) \in \hat{}(C \rightarrow A)$ iff for all i', if $(j, i') \in C$, then $(i, i') \in \hat{}A$.

Let me abbreviate $\hat{}(C \rightarrow A)$ by $\hat{}CA$. It is not difficult to prove that, in the particular case when, for all i, j in Mp, $\hat{}A(i) = \hat{}A(j)$, it holds that for all i, j in Mp $\hat{}CA(i) = \hat{}CA(j)$; $\hat{}CA$ is a standard law if $\hat{}A$ is a standard law too.

3 A standard law is valued at a certain point (a potential model) i; but, to value a constraint, a pair (i, j) of points is needed, and we say that constraint C is true at (i, j). The basic claim is this: the phrase "X is true at the pair (i, j) of points" is equivalent to: "X is true at point g which is endowed with an inner structure". This statement is, at first, puzzling, for the switch seems to be only a verbal one. If a pair (i, j) of potential models were identified with a point g, nothing, indeed, seemed to have been gained. What is meant then is this. The notion of "valuation point" is ambiguous. Roughly speaking, a valuation point can be specified either as a potential model (with perhaps some additional properties) or as a pair of potential models.
I agree that such a specification presents great difficulties. Fortunately, some hope comes from Segerberg's work. In Segerberg (1973) he introduced in modal logic the notion of "diagonal" and related concepts. It comes then possible to show that each potential model could be represented as a pair of remarcable potential models of the theory. The set of all these remarkable Mp's is called the diagonal DMp of Mp. Each potential model i could then be represented as a pair (j, j') of elements in DMp. If i is in DMp, then the corresponding pair is (i, i). Now it throws new light on the significance of constraints: if some potential model i is not on the diagonal, then it could not be represented as (i, i), but as (j, j'), with j, j' in DMp. Therefore, some connections among MP's are needed (of course, only in so far as one agrees that there are potential models outside of the diagonal of Mp). If $DMp \neq Mp$, constraints are not trivial (i.e. $C \neq Mp \times Mp$) and therefore the transport of information across potential models is not total. This problem and other topics concerning DMp are discussed at length in Miroiu (1984a).

In modal logic one is usually concerned with propositions $\hat{}A$ (sets of possible worlds) and with an alternativeness relation R. He needs to produce modal propositions such as, e.g., the proposition expressed by the sentence "It is necessary that A". Those propositions $\hat{}RA$ have to be constructed as sets of possible worlds. The standard account is this: $w \in \hat{}RA$ iff for all w', if $(w, w') \in R$, then $w' \in \hat{}A$. Example (C. I. Lewis's system S5): if $\hat{}A = K$, then $\hat{}RA = K$; otherwise $\hat{}RA = \emptyset$.

Both in the general and also in the particular case I mentioned above, it is possible to show that, if C is reflexive and symmetrical, the following hold:

B1. $\hat{}CA \subset \hat{}A$

B2. $\hat{}C(A \cap B) = \hat{}CA \cap \hat{}CB$

B3. $\hat{}\overline{C\overline{CA}} \subset \hat{}A$ (if $\hat{}B$ is a T-proposition, then $\hat{}\overline{B}$ is $Mp \times Mp - \hat{}B$)

B4. $\hat{}C(Mp \times Mp) = Mp \times Mp$.

My claim is that the notion of a *generalized law* is a good substitute for Sneed's dichotomy law-constraint. However, many interesting uses of constraints involve transporting information across different potential models of the theory. The problem, Sneed argued in correspondence, is exactly this: how my conception of constraint as generalized law could be used to treat these cases.

A short glance at modal logic seems to be instructive. Suppose $V(A, w)$ and relation R are already defined. Then it is possible to define $V(\Box A, w)$ as shown in section 1. Suppose, on the other hand, that we know, for all A and all $w \in K$ if $V(A, w) = 1$ or $V(A, w) = 0$ or if $V(A, w)$ is not defined. Then it is possible to give an adequate definition of relation R if we require that:

$R(w, w')$ iff $V(A, w') \neq 0$ for all A such that $V(\Box A, w) = 1$. (Henceforth I assume that $\hat{}A$'s are standard laws, i.e. for all i and j in Mp, $\hat{}A(i) = \hat{}A(j)$.) Let i, j be potential models of the theory in which generalized laws hold. Now constraints could be reconstructed as cross-connexions between i and j. The definition I suggest is this:

A transport of information is involved across i and j (i.e. $C(i, j)$ holds) iff for all $\hat{}A$, if $i \in \hat{}CA$, then $j \in \hat{}A$.

It is not difficult to prove relation C just defined is reflexive and symmetrical. The first property yields immediately from Bl. Second, prove that if $C(i, j)$, then $C(j, i)$:

1. if $i \in \hat{}CA$, then $j \in A$ for all $\hat{}A$ (supposition: $C(i, j)$)
2. if $i \in \hat{}C\overline{CA}$, then $j \in \hat{}\overline{CA}$ (by substituting $\hat{}\overline{CA}$ for $\hat{}A$ in (1))
3. if $i \in \hat{}\overline{C\overline{CA}}$, then $i \in \hat{}A$ (from B3)
4. if $i \in \hat{}\overline{A}$, then $i \in \hat{}C\overline{CA}$ (by transposition from (3))
5. if $i \in \hat{}\overline{A}$, then $j \in \hat{}\overline{CA}$ (from (2) and (4))

6. if $j \in {}^\wedge CA$, then $i \in {}^\wedge A$ (by transposition from (5))
7. $C(j, i)$ (by the definition of relation C) q.e.d.

To conclude: if one starts with $^\wedge A$'s and $^\wedge CA$'s, then constraint C can be reconstructed as a relation on *Mp*. Provided that $^\wedge A$'s and $^\wedge CA$'s are settled at a potential model i, it follows that i can tell us something about other potential models j, j', j'' ... But there is another problem. Sneed holds that there are good reasons to put apart constraints from laws. Laws hold *in* some intended application of a theory and describe its specific features; on the other hand, constraints do not represent a feature of some intended application, but rather a certain kind of connection obtaining between all intended applications. The formal counterpart of this idea consists in taking laws as sets of *Mp*'s and constraints as relations on *Mp*. Sneed claims, however (in Sneed 1977), that his treatment lies on an intuitive idea.

Suppose now that one would argue as follows: you tried to show that constraints on *T*-non-theoretical functions are elements in the very structure of each potential model. They could therefore be identified with trivial constraints $C' = Mp \times Mp$. Then, for all $^\wedge A$, $^\wedge A = {}^\wedge CA$ holds and thus if $X \subseteq Mp$, it is possible to view it either as an $^\wedge A$-law, or as a "modal" $^\wedge CA$-law. But that is exactly what makes my point: to draw an intuitive distinction between $^\wedge A$ and $^\wedge CA$ one has to account constraints C on some *T*-theoretical function, of which $^\wedge A \neq {}^\wedge CA$ does not in general hold.

And yet the challenge needs to be faced. My reply would be satisfactory if certain *tests* are provided so that they would count as evidence either for $^\wedge A$ but not for $^\wedge CA$, or for $^\wedge CA$ but not for $^\wedge A$. These tests need to be "empirical" or at least be dependent on some other theory T' (on which certain restrictions are imposed). This argument is misleading, however: for if indeed some systematic means to distinguish $^\wedge A$'s from $^\wedge CA$'s are required, it is not me who is in the position to offer them. I do not think that standard laws differ from (standard) constraints more than $^\wedge A$'s differ from $^\wedge CA$'s. If there is any difference at all between a "natural law" such that "$f = m \cdot a$" and a "natural law" as "Mass is an extensive quantity", then there it is also some difference between "standard" laws ($^\wedge A$-laws) and "modal" laws ($^\wedge CA$-laws). In so far as some reasons to divorce $^\wedge A$ from C – or, equivalently, $^\wedge A$ from $^\wedge CA$ – are required, it is Sneed who has to bear the burden of the argument.

On the other hand, it still seems to me that the use of $^\wedge A$'s and $^\wedge CA$'s, rather than the use of laws and constraints (in Sneed's sense) is somehow more intuitive. Indeed, I believe that the logical behaviour of $^\wedge A$ is quite different from the logical behaviour of $^\wedge CA$. Logicians have at length discussed tests for "modal phrases" and argued that some modal contexts are involved at least in cases when substitution (*salva veritate*) and/or existential generalization fail. Naturally, one would expect that such tests could be designed when a certain theory, e.g. *CPM*, is taken into account (that is main subject in Miroiu (1984a).

A well-known (and more general) modal distinction is still at hand. It is that between *de dicto* and *de re* modalities. Suppose George thinks now of number 7. Then: "The number George thinks now about is necessarily prime" is a *de re* (and true) sentence; but "Necessarily, the number George thinks now about is prime" is a *de dicto* (and false) sentence, for there is no necessity that George would think now of a prime number. Now, I believe that the law-constraints (or, equivalently, $^\wedge A$-law – $^\wedge CA$-laws) dichotomy roughly falls over the modal *de dicto* – *de re* one. Think of the following two sentences. The first expresses a standard $^\wedge A$-law: "Necessarily, for each particle x the ratio f/a equals its mass m". The second sentence expresses the corresponding $^\wedge CA$-law (when the only constraint C on m was supposed to be independence of system): "For each particle x with mass $m(x) = k$, necessarily $f = k \cdot a$" (Note that according to Sneed "necessarily" should be understood as: "in all intended applications").

Of course, one might argue that the *de re* and even the *de dicto* modalities meet an enormous realm of ghosts and mysteries. Perhaps. But it was not my aim to dissolve it here: for I have only tried to bring about modal counterparts of some of Sneed's basic concepts.

Let us turn back to the notion of a T-law. We have two rival definitions for this concept. The weak one is this:

X is a weak T-law iff: 1) X as a generalized law; and 2) for each $i \in M$, $(i, i) \in X$.

The strong definition is the following:

X is a strong T-law iff: 1) X is a generalized law; and 2) for all i, j in M, $(i, j) \in X$.

The strong definition is the natural generalization of the standard one I mentioned at the beginning of this section. Nevertheless, the weak concept of a T-law is itself interesting. First, one needs not specify pairs of two different models of T. He needs only pairs like (i, i), (j, j)[4] ... Second, though each generalized law X presupposes the existence of constraint C, X's being a weak T-law involves only pairs like (i, i) and not like (i, j). Thus, it does not appeal at cross-connexions between i and some (other) potential model j. That is an additional manner to confine constraints inside (potential) models of T.

4 In Miroiu (1984a) I try to argue that such pairs of potential models define points on the diagonal *DMp* of *Mp*.

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